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Some coincidence point results for generalized (ψ,φ)-weakly contractions in ordered b-metric spaces

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Abstract

In this paper we present some coincidence point results for four mappings satisfying generalized (ψ,φ)-weakly contractive condition in the framework of ordered b-metric spaces. Our results extend, generalize, unify, enrich, and complement recently results of Nashine and Samet (Nonlinear Anal. 74:2201-2209, 2011) and Shatanawi and Samet (Comput. Math. Appl. 62:3204-3214, 2011). As an application of our results, periodic points of weakly contractive mappings are obtained. Also, an example is given to support our results.

MSC: 47H10; 54H25

Keywords: b-metric space; partially ordered set; fixed point; altering distance function

1 Introduction

A self-mapping f on a metric space (X,d) is a contraction, if $d(fx,fy)\leq kd(x,y)$ for all $x,y\in X$, where $k\in[0,1)$.

The Banach contraction principle, which shows that every contractive mapping defined on a complete metric space has a unique fixed point, is one of the famous theorems which was generalized by many researchers in different ways [1–5] and [6–12].

A self-mapping f on X is a weak contraction, if $d(fx,fy)\leq d(x,y)-\varphi(d(x,y))$ for all $x,y\in X$, where φ is an altering distance function.

The above concept was introduced by Alber and Guerre-Delabriere [13] in the setup of Hilbert spaces. Rhoades [14] generalized the Banach contraction principle by considering this class of mappings in the setup of metric spaces and proved that every weakly contractive mapping defined on a complete metric space has a unique fixed point.

Let f and g be two self-mappings on a nonempty set X. If $x=fx=gx$ for some x in X, then x is called a common fixed point of f and g.

Zhang and Song [15] introduced the concept of a generalized φ-weak contractive mappings and proved the following common fixed point result.

Theorem 1 [15] *Let (X,d) be a complete metric space. If $f,g:X\to X$ are generalized φ-weak contractive mappings, then there exists a unique point $u\in X$ such that $u=fu=gu$.*

For further work in this direction, we refer to [1, 16, 17] and [18].